TIME-RECURSIVE SOLUTION TO THE INVERSE PROBLEM OF ELECTROCARDIOGRAPHY: A MODEL-BASED APPROACH

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Abstract

This communication presents a new approach to the reconstruction of epicardial potentials from measured body surface potentials. Its salient characteristic is that the temporal evolution of the epicardial potentials is accounted for by means of an *explicit* model. The type of the model is chosen so as to yield efficient reconstruction using Kalman filtering. Classical Tikhonov regularization appears as a special case of the resulting technique, which can also be viewed as a generalization of the work presented in [1]. Results obtained on real data and comparison with those produced by existing methods confirm the interest of the approach.

1 Introduction

Our goal is to reconstruct epicardial potentials (EPs), whose temporal and spatial characteristics are of interest, from measured body surface potentials. Since the corresponding inverse problem is ill-posed, it must be regularized through incorporation of prior information on the solution. Tikhonov regularization, which presents the advantage of providing a solution in closed form, has been widely used [2]. However, in this approach, the time-continuity of the cardiac activation process is not accounted for as the data are independently processed time-frame by time-frame. Oster & Rudy [1] assessed the importance of temporal information and found it could improve the results significantly. However, practical use of temporal information remains an open question. In our approach, the time-dependence across time-frames is accounted for through a linear prediction model, which allows us to reconstruct the EPs using a Kalman filter.

2 Methods

The relationship between body surface potentials and EPs can be considered linear with a good approximation [3]. Therefore, at time-sample k, one can write

$$\boldsymbol{\Phi}_T(k) = \boldsymbol{H}\boldsymbol{\Phi}_E(k) + \boldsymbol{n}(k), \qquad (1)$$

where $\boldsymbol{\Phi}_T$ and $\boldsymbol{\Phi}_E$ respectively denote the vectors of body surface potentials and EPs, and where $\boldsymbol{H}$ is the transfer matrix between the heart and the torso; $\boldsymbol{n}$ is a Gaussian white noise vector which represents modeling and measurement errors. In order to account for the time-correlation between EPs, the following time-invariant linear prediction model is introduced:

$$\boldsymbol{\Phi}_E(k+1) = \boldsymbol{F}\boldsymbol{\Phi}_E(k) + \boldsymbol{\epsilon}(k+1), \qquad (2)$$

where $\boldsymbol{F}$ and $\boldsymbol{\epsilon}$ denote the time-invariant prediction matrix and the prediction error vector, respectively. (2) and (1) form a state-space representation of the phenomena, which allows us to compute the least mean squares estimates of the EPs with the following Kalman filter (expressed here in compact form):

$$\widehat{\boldsymbol{\Phi}}_E(k+1 \mid k) = \boldsymbol{F}\widehat{\boldsymbol{\Phi}}_E(k \mid k-1) + \boldsymbol{K}_k\boldsymbol{R}_k^{-1}\left(\boldsymbol{\Phi}_T(k) - \boldsymbol{H}\widehat{\boldsymbol{\Phi}}_E(k \mid k-1)\right) \qquad (3)$$

$$\boldsymbol{K}_k \triangleq \boldsymbol{F}\boldsymbol{P}_{k|k-1}\boldsymbol{H}^T \qquad (4)$$

$$\boldsymbol{R}_k \triangleq \boldsymbol{R}^n + \boldsymbol{H}\boldsymbol{P}_{k|k-1}\boldsymbol{H}^T \qquad (5)$$

$$\boldsymbol{P}_{k+1|k} = \boldsymbol{F}\boldsymbol{P}_{k|k-1}\boldsymbol{F}^T - \boldsymbol{K}_k\boldsymbol{R}_k^{-1}\boldsymbol{K}_k^T + \boldsymbol{R}^\epsilon. \qquad (6)$$

In (3)-(6), $\boldsymbol{R}^n$ and $\boldsymbol{R}^\epsilon$ denote the covariance matrices of processes $\boldsymbol{n}$ and $\boldsymbol{\epsilon}$. These matrices, along with the initial values of the estimate $\widehat{\boldsymbol{\Phi}}_E(1 \mid 0)$ and of its covariance matrix $\boldsymbol{P}_{1|0}$, control the behavior of the Kalman filter.

At this point, two major problems need to be solved: (i) determination of the state-space model (2)-(1), i.e., $\boldsymbol{H}$ and $\boldsymbol{F}$; (ii) specification of the quantities which control the Kalman filter. Regarding point (i), determination of $\boldsymbol{H}$ has been widely studied. Here, this matrix was computed using a 3-D finite element model of the torso according to the technique described in [3]. Determination of $\boldsymbol{F}$ is less classical, and we chose to estimate its value from recorded EPs. This estimation problem is also ill-posed, and therefore must be regularized. As the degree of accuracy of the prediction model necessary to achieve satisfactory reconstruction was difficult to assess beforehand, two types of regularizing constraints were used. 1) $\boldsymbol{F} = a\boldsymbol{I}$, where a was estimated using a least-

squares method. 2) Determination of $\boldsymbol{F}$ using a regularized least-squares approach. The regularizing matrix was selected so as to make the predicted potentials at a given electrode depend mostly on its neighbors. The estimator was implemented using a recursive least-squares algorithm. In order to solve point (ii), the prediction error was analyzed so as to determine the corresponding covariance matrix $\boldsymbol{R}^{\epsilon}$. Setting the initial time just before cardiac activation allowed us to initialize the Kalman filter with $\widehat{\boldsymbol{\Phi}}_E(1 \mid 0) = \mathbf{0}$ and $P_{1|0} = \alpha \boldsymbol{I}, \alpha \ll 1$. Finally, $\boldsymbol{n}$ was assumed uncorrelated spatially which gives $\boldsymbol{R}^n$ a diagonal structure. Its entries were specified heuristically according to the observation signal-to-noise ratio (SNR).

3 Results

EPs were measured with a 63-lead mapping system using a sock electrode array in 8 patients with Wolff-Parkinson-White syndrome who had undergone arrhythmia surgery. Each recording was taken during normal sinus rhythm and lasted 1.024 s. In order to assess the generality of the prediction model, $\boldsymbol{F}$ was identified for each patient separately and used for predicting the EPs of the other patients, for both types matrices $\boldsymbol{F}$. The results were compared using a relative squared error (RSE) criterion.

For general matrices $\boldsymbol{F}$, estimation and cross-patient RSEs ranged from 0.01 to 0.25. and from 0.38 to 0.80, respectively. This indicates that $\boldsymbol{F}$ is specific to each patient. For $\boldsymbol{F} = a\boldsymbol{I}$, estimation and cross-patient RSEs were in the intervals $[0.37, 0.50]$ and $[0.20, 0.60]$ respectively, which shows that this prediction model is less accurate, but more widely usable than the previous one.

Body surface potentials were simulated by multiplying EPs with transfer matrix $\boldsymbol{H}$ and adding Gaussian noise to the result. Two complete data sets were generated, with respective SNR values of infinity and 20 dB. Then reconstruction of the EPs was carried out using Kalman filter (3)-(6). Patient-specific prediction models were used, both for general matrices $\boldsymbol{F}$ and for $\boldsymbol{F} = a\boldsymbol{I}$. $\boldsymbol{R}^n$ was set as though the SNR value were 100 dB for the first data set, and 10 dB for the second one. For the sake of comparison, two other reconstruction methods were implemented: classical Tikhonov regularization, and Twomey regularization using $\boldsymbol{\Phi}_E(k-1)$ as the estimate of the solution [1]. In these cases, the regularization parameter was determined so as to minimize the reconstruction RSE. The results are collected in Table 1. Best reconstructions were obtained using our approach with a general matrix $\boldsymbol{F}$.

Patient	Kalman $\boldsymbol{F} = a\boldsymbol{I}$	Kalman Gen. $\boldsymbol{F}$	Tikhonov 0-order	Twomey $\boldsymbol{\Phi}_E(k-1)$
		SNR = ∞		
1	0.0378	0.0067	0.0113	0.0084
2	0.0742	0.0083	0.0101	0.0119
3	0.0164	0.0045	0.0043	0.0034
4	0.0256	0.0049	0.0019	0.0006
5	0.0282	0.0055	0.0097	0.0035
6	0.0311	0.0026	0.0094	0.0020
7	0.0748	——	0.0085	0.0061
8	0.0397	0.0050	0.0033	0.0013
		SNR = 20 dB		
1	0.3789	0.0824	0.3623	0.2379
2	0.4836	0.1240	0.4596	0.3848
3	0.4289	0.0761	0.4066	0.3074
4	0.3353	0.0755	0.3748	0.1749
5	0.2853	0.0790	0.3677	0.1775
6	0.3671	0.0651	0.3095	0.1331
7	0.3632	——	0.3970	0.3110
8	0.4444	0.1080	0.4046	0.3035

Table 1: Reconstruction RSEs

4 Discussion and conclusion

The above results illustrate the interest of our approach, which produced low reconstruction errors. Clearly, the critical point is the selection of an appropriate prediction model. As $\boldsymbol{F}$ is patient-specific, the most satisfactory approach would be to estimate its value *directly from body surface potentials*. Techniques developed for blind estimation could be used for this purpose, but simplification of the structure of $\boldsymbol{F}$ would certainly be required in order to reduce the number of unknown parameters. Note that the same techniques could also be used for determination of the diagonal entries of $\boldsymbol{R}^n$. Other potential improvements are the use of a Kalman *smoother* so as to further reduce the reconstruction RSE, and utilization of fast or asymptotic Kalman filters in order to decrease the computational load.

References

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